Ph.D. Course work

Pre-Ph.D. Examination Syllabus

DEPARTMENT OF MATHEMATICS,
K L UNIVERSITY,
VADDESWARAM - 522502, ANDHRA PRADESH, INDIA.

KL UNIVERSITY
Green Fields, Vaddeswaram.

List of Pre-Ph.D Courses approved by

DEPARTMENT OF MATHEMATICS

S.NO	PAPER – 2	Code	PAPER – 3	Code
1.	Topology	15 MAT 201	Fluid Dynamics	15 MAT 301
2.	Numerical Methods	15 MAT 202	Fluid Mechanics	15 MAT 302
3.	Time Scale Calculus	15 MAT 203	Tribology of Bearings	15 MAT 303
4.	Number Theory	15 MAT 204	Fuzzy Algebra	15 MAT 304
5.	Special Functions	15 MAT 205	Functional Analysis	15 MAT 305
6.	Boundary Value Problems	15 MAT 206	Semi Groups	15 MAT 306
7.	Distribution and Estimation Theory	15 MAT 207	Dynamical Systems on Time scale	15 MAT 307
8.	Mathematical methods & Stochastic processes	15 MAT 208	Differential Equations	15 MAT 308
9.			Cryptography	15 MAT 309
10.			Hyper Geometric Functions and Lie groups	15 MAT 310
11.			Queueing Theory	15 MAT 311
12.			Sampling Theory	15 MAT 312
13.			Differential Geometry	15 MAT 313
14.			Inventory Model	15 MAT 314
15.			Difference Equations	15 MAT 315

Code: 15 MAT 201

DEPARTMENT OF MATHEMATICS

TOPOLOGY
SYLLABUS

Unit –I Topological Spaces and Continuous Functions

Topological spaces, basis for a topology, the order topology, the product topology on X x Y, the sub space topology, closed sets and limit points, continuous functions, the product topology, the metric topology.

Unit –II Connectedness and compactness

Connected spaces, connected subspaces of the real line, compact spaces, compact subspaces of the real line, limit point compactness.

Unit –III Countability and separation axioms

The countability axioms, the separation axioms, normal spaces, the urysohn lemma, the urysohn metrization theorem.

Unit –IV The Tychnoff Theorem

The Tychnoff Theorem, Completely Regular Spaces, The Stone –Cech Compactification.

Unit –V Complete metric spaces and function spaces

Complete metric spaces, compactness in metric spaces, pointwise and compact convergence, ascoli's theorem.

Note: 1. 8 Questions to be set out of which 5 Questions to be answered.
2. Questions should be uniformly distributed from all the units.

Prescribed text Book:
1. Topology by **James Dugundji**; Universal Book Stall, New Delhi.

2. Introduction to Topology by **G.F.Simmons**; Tata McGraw-Hill Publishing Company.

Reference Text Book:
1. Topology by **James R.Munkres**; Prentice-Hall, Second edition.

TOPOLOGY

MODEL PAPER

Time: 3 hour **Max Marks:100**

Note: Answer ANY FIVE from the following.

1. (a) Describe the lower limit topology T in the set R of real numbers. Is T finer than the usual topology on R? Justify.

(b) Suppose B and B' are base for topologies T and T' on a set X. If every is a subset of some? Justify.

2. (a) Describe the dictionary topology on R X R and prove that this topology coincides with the product topology where is R equipped with the discrete topology and the second factor R has the usual topology.

(b) Show that for a subset A of X, = A U A**I**Model Question Paper, Mathematics paper

3. (a) Show that the Cartesian product of connected spaces is connected.

(b) Give an example of a connected space which is not path connected.

4. (a) Show that a metrizable space X is compact if and only if X is sequentially compact.

(b) Show that the Cantor set is compact.

5. Which of the following are true? Justify

(i) If X and Y are second countable so is X X Y

(ii) If X and Y are Lindelof spaces so is X X Y

6. (a) Show that every regular space with a countable basis is normal.

(b) Show that a connected normal space having more than one element is normal.

7. (a) S.T A metric space X is complete iff every Cauchy's sequence in X has a convergent sequence.

(b) If X is a complete topological space ,Show that the space C(X,R) of all continuous real valued functions on X is complete under the metric C defined by C(f,g)={|f(x)-g(x)|}Model Question Paper, Mathematics paper

8. S.T a metric space X is compact iff X is complete and totally bounded.

Code: 15 MAT 305

FUNCTIONAL ANALYSIS

SYLLABUS

Unit-1 Linear Metric Spaces

Vector Spaces, Linear Metric Spaces, Normed Linear Spaces.

Unit-2 Basic Theorems On Normed Linear Spaces

Bounded Linear Transformations, Hahn-Banach Theorem, Open Mapping Theorem, Banach – Steinhaus Theorem.

Unit-3 Hilbert Spaces

Inner Product Spaces, Orthonormal Sets, Riesz Representation Theorem, Bounded Linear Operations On Hilbert Spaces.

Unit-4 Fixed Point Theory

The Contraction Mapping Theorem And Its Applications, Brouwer's Fixed Point Theorem And Its Applications, Schauder's Fixed Point Theorem And Some Related Results.

Unit-5 Partial Metric Spaces

Definitions Some Examples, Banach Fixed Point Theorem, $\psi-\phi$ Contraction Theorem For Four Maps And Corollaries Of This Theorem, Suziki Type Fixed Point Theorem For Single Valued Maps, W- Comparability, A Unique Common Coupled Fixed Point Theorem For Four Maps.

Prescribed Text Book:

Functional Analysis With Applications By **B.Choudhary** And **Sudarsan Nanda**; Wiley Eastern Limited.

FUNCTIONAL ANALYSIS

MODEL PAPER

Time: 3 hour **Max Marks:100**

Note: Answer <u>ANY FIVE</u> from the following.

1. (a) state and prove Riesz Lemma

(b) prove that c[a,b] is a Normed Linear Spaces with ‖ f ‖ =sup | f(x) |

2. (a) state and prove Open Mapping Theorem.

(b)Let E be a real normed linear space and let M be a linear subspace of E if $f\in M^*$, then there is a $g\in E^*$ such that $f\subset g$ and ‖g‖=‖f‖.

3.(a) state and prove Riesz representation theorem .

(b) if{ e1,e2.............en} is a finite orthonormal set in an inner product space X and x is any element of X, then

$$\sum_{i=1}^{n} |(x, ei)|2 = \|xxxxxxxxx\|2 \text{ and } \quad x- \quad \sum_{i=1}^{n}(x, ei)ei \perp ej$$

(j=1,2,........,n)

4.(a) state and prove banach fixed point theorem.

(b) consider the following fredholm integral equation

$$f(x)=g(x)+\lambda\int_0^1 k(x,y)\ f(y)\ dy$$

where $g\in L2[0,1]$ and $k\in L2([0,1]\times[0,1])$ prove that if g=0 implies f=0 then there exists a unique solution of the equation for any $g\in L2[0,1]$

5. state and prove picard's theorem.

6. *Let* (X, p) *be a partial metric space and let* $S, T, f, g : X \to X$ *be such that*
$\psi(p(Sx, Ty)) \le \psi(M(x, y)) - \phi(M(x, y))$, $\forall\ x, y \in X$,
where $\psi : [0,\infty) \to [0,\infty)$ *is continuous , non-decreasing and* $\phi : [0,\infty) \to [0,\infty)$ *is lower semi continuous with* $\phi(t) > 0$ *for* $t > 0$ *and*
$M(x, y) = \max\{ p(fx, gy), p(fx, Sx), p(gy, Ty), 1/2\ [p(fx, Ty) + p(gy, Sx)] \}$
$S(X) \subseteq g(X)$, $T(X) \subseteq f(X)$
either $f(X)$ *or* $g(X)$ *is a complete subspace of* X *and the pairs* (f, S) *and* (g, T) *are weakly compatible.*
Then S, T, f and g have a unique common fixed point in X.

7. Let (X, p) be a partial metric space and let $f, g : X \to X$ and $F,G : X \times X \to X$ be such that
(i) For all $x, y, u, v \in X$,
$\psi(p(F(x, y),G(u, v))) \le$
$1/2\ \psi(p(fx, gu) + p(fy, gv)) - \varphi(p(fx, gu) + p(fy, gv))$,

where $\psi \in \Psi$ and $\varphi \in \Phi$,
(ii) $F(X \times X) \subseteq g(X), G(X \times X) \subseteq f(X)$,
(iii) either $f(X)$ or $g(X)$ is a complete subspace of X and
(iv) the pairs (F, f) and (G, g) are w - compatible.
Then F, G, f and g have a unique common coupled fixed point in $X \times X$. Moreover, the common
coupled fixed point of F, G, f and g have the form (u, u).

8.(a)show that □□ is a vector space with usual coordinate wise addition and scalar multiplication.

(b) show that the set of all real-valued functions of real variables is a vector space with the usual point –wise addition and scalar multiplication.

Code: 15 MAT 202

NUMERICAL METHODS
SYLLABUS

UNIT-I

Numerical Differentiation and Integration

Introduction, Numerical Differentiation, Numerical Integration, Euler-Maclaurin Formula, Adaptive Quadrature Methods, Gaussian Integration, Singular Integrals, Fourier Integrals, Numerical Double Integration

UNIT-II

Numerical Solution of Ordinary Differential Equations

Introduction, Solution by Taylor's Picard's Method, Euler's Method, Runge-Kutta Methods, Predictor-Corrector Methods, the Cubic Spline Method, Simultaneous and Higher Order Equations, Boundary Value Problems: Finite-Difference Method, The Shooting Method,

UNIT-III

Numerical Solution of Partial Differential Equations

Introduction, Finite-Difference Approximations, Laplace's Equation: Jacobi's Method, Gauss-Seidel Method, SOR Method, ADI Method, Parabolic Equations, Iterative Methods, Hyperbolic Equations.

UNIT-IV

System of Linear Algebraic Equations

Introduction, Solution of Centro-symmetric Equations, Direct Methods, LU- Decomposition Methods, Iterative Methods, III-conditioned Linear Systems.

UNIT-V

The Finite Element Method: Functionals- Base Function Methods of Approximation- The Rayleigh –Ritz Method –The Galerkin Method, Application to two dimensional problems-Finite element Method for one and two dimensional problems.

Reference Books:

1. Niyogi, Pradip, "Numerical Analysis and Algorithms", Tata McGraw –Hill
2. Balagurusamy,E., "Numerical Methods", Tata McGraw –Hill
3. Sastry, S.S., "Introduction Methods of Numerical Analysis", PHI
4. Chapra, S.C. and Canale, R.P., "Numerical Methods for Engineers", Tata McGraw –Hill

NUMERICAL METHODS
MODEL PAPER

TIME: 3Hrs **Marks: 100**

Answer any five from the following **5X20=100**

1. (a) Using R-K method of forth order, find $y(0.1)$ and $y(0.2)$ for the equation

$$\frac{dy}{dx}=\frac{y-x}{y+x}, y(0)=1, \text{take } h=0.2.$$

(b) Solve the equation $y''=x+y$ with the boundary conditions $y(0)=y(1)=0.$

2. (a) The deflection of a beam is governed by the equation $\frac{d^4y}{dy^4}+81y=f(x)$, where $f(x)$ is given by the table

x	1/3	2/3	1
$f(x)$	81	162	243

and boundary condition c. Evaluate the deflection at the pivotal of the beam using three sub-intervals.

(b) Using Picard's method find an approximate values of y and z corresponding to $x=0.1$, given that $y(0)=2, z(0)=1$ and $\frac{dy}{dx}=x+z, \frac{dz}{dx}=x-y^2.$

3. (a) Solve the Laplace equation for the square mesh of the following figure with boundary values as shown.

	500	1000	500	
		C		
A	u_1	u_2	u_3	
	u_4	u_5	u_6	B
	u_7	u_8	u_9	
		D		
	500	1000	500	

(b) Find the solution of the initial boundary value problem $\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$, $0\le x\le 1;$ subject to the initial conditions $u(x,0)=\sin \pi x, 0\le x\le 1;$ and the boundary conditions $u(0,\mathrm{t})=u(1,t)=0, \mathrm{t}>0$, by using in the (i) the explicit scheme and (ii) the implicit scheme.

4. (a) Use adaptive quadrature to evaluate the integral $\int_{0.1}^{2}\sin\frac{1}{x}\,dx$ to within an accuracy $\varepsilon=0.001$

(b) Use 3-point Gauss – Legendre formula to evaluate the integral $\int_{0}^{\pi/2}\sin x\,dx$.

5. (a) Use Rayleigh – Ritz method to solve the BVP $\frac{d^2y}{dx^2}+2x=0,\ y(0)=y(1)=0$

(b) Using Galerkin method, solve Poisson's equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\mathrm{k},\ 0<x,\ y<1$ with $u=0$ on the boundary C of the region R.

6. (a) Using shooting technique, solve the BVP
$$y''(x)-y(x)=0;\ y(0)-0,\ y(1)=1.1752,\text{we choose } m_0=0.7, m_1=0.8.$$

(b) Given the BVP $x^2y''+xy'-y=0,\ y(1)=1, y(2)=0.5$ apply the cubic spline method to determine the value of $y(1.5)$

7. Decompose the matrix
$A=\begin{bmatrix}5 & -2 & 1\\ 7 & 1 & -5\\ 3 & 7 & 4\end{bmatrix}$ into the form LU and hence solve the system AX=B where B=[4 8 10]^T.

Also Determine L^{-1} and U^{-1} and hence find A^{-1}

8. (a) Solve the heat conduction equation $\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial y^2}$, subject to the boundary conditions $u(0,t)=u(1,t)=0$ and $u(x,0)=x-x^2$, take $h=0.25$ and $k=0.025.$

(b) Determine the solution of the following system of linear equations

$$\begin{aligned}20x + y - 2z &= 17\\ 3x + 20y - z &= -18\\ 2x - 3y + 20z &= 25\end{aligned}$$

using Jacobi method.

Code: 15 MAT 206

KL UNIVERSITY
DEPARTMENT OF MATHEMATICS
Pre Ph.D. Examinations
BOUNDARY VALUE PROBLEMS

Eight questions are to be set and the student has to answer five in three hours of duration:

UNIT-1 : **System of linear differential equations:** system of first order equations, existence and uniqueness theorem, fundamental matrix, non-homogeneous linear systems, linear systems with constant coefficients.

Unit-2 : **Existence and Uniqueness of Solutions:** introduction, preliminaries, successive approximations, Picard's theorem, continuation and dependence on initial condition, existence of solutions in the large interval.

(Scope and treatment as in chapters: 4 and 5 of Text book (1))

Unit-3: **Nonlinear boundary value problems:** Kinds of boundary value problems associated with Non-linear second order differential equations, generalized Lipschitz condition, failure of existence and uniqueness of linear boundary value problems, simple nonlinear BVP, standard results concerning initial value problems.

Unit-4: **Relation between the first and second boundary value problems:** relation between uniqueness intervals, relation between existence intervals.

Unit-5: **Contraction mapping:** introduction, Contraction mappings, boundary value problems, a more generalized Lipschitz condition.

(Scope and treatment as in chapters: 1, 2 and sections 3.1 to 3.4 of chapter 3 of Text book (2))

Text Books:

1. Text book of ordinary differential equations by S. G. Deo, V. Lakshmikantham and V. Raghavendra, Second edition, Tata McGraw-Hill Publishing Company Ltd, New Delhi (2002).
2. Non-linear two point boundary value problem by P. B. Bailey, L. P. Shampine and P. E. Waltman, Academic press, New York and London (1968).

KL UNIVERSITY
DEPARTMENT OF MATHEMATICS
Pre-Ph.D Degree Examination
MODEL QUESTION PAPER
BOUNDARY VALUE PROBLEMS

Time: 3Hours **Max.Marks:100**

Answer any FIVE questions from following, each question carries equal marks.

1. (a) Find the fundamental system of solutions for the system of equations

$$x_1'(t) = x_1(t), \quad x_2'(t) = 2x_2(t), \text{ for all } t \in [0,1].$$

(b) Compute the solution of the following non-homogeneous system $x' = Ax + b(t)$, where

$$A = \begin{bmatrix} 3 & 2 \\ 0 & 3 \end{bmatrix}, b(t) = \begin{bmatrix} e^t \\ e^{-t} \end{bmatrix} \text{ and } x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

2. (a) Determine exp(At) for the system $x' = Ax$, where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -11 & 6 \end{bmatrix}.$$

(b) Prove that if x(t) is the solution of the initial value problem $x' = f(t,x), x(t_0) = x_0$ on some interval if and only if x(t) is solution of the corresponding integral equation.

3. (a) If f(t, x) is continuous function on $|t| \le \infty$, $|x| < \infty$ and satisfies Lipschitz condition on the strip S_a for all a > 0, where Sa = {(t, x) : $|t| \le a$, $|x| < \infty$}. Then show that the initial value problem $x' = f(t,x), x(t_0) = x_0$ has a unique solution existing for all t.

(b) State and prove Picard's theorem.

4. Define the kinds of boundary value problems and discuss the solution of nonlinear boundary value problem $y''(t) + |y(t)| = 0$, satisfying $y(0) = 0, y(b) = B$.

5. (a) Let a < c < b. If uniqueness hold for all second boundary value problems $y''(t) + f(t, y(t), y'(t)) = 0$, $y(a) = A, y'(c_1) = m$, whenever $c_1 \in [a,c]$ and if uniqueness hold for all second boundary value problems $y''(t) + f(t, y(t), y'(t)) = 0$, $y'(c_1) = m, y(b) = B,$ whenever $c_1 \in [c,b]$, then show that uniqueness holds for all first boundary value problems $y''(t) + f(t, y(t), y'(t)) = 0$, $y(a) = A, y(b) = B.$

(b) Obtain the Green's function for second order equation $y''(t)+F(t,y(t),y'(t))=0,$ satisfying the zero boundary conditions $y(a)=0, y(b)=0$,

And also find bounds for Green's function.

6. (a) Let $f(t,y)$ be continuous on $[a,b]\times(-\infty,\infty)$ and satisfies $|f(t,x)-f(t,y)|\le K|x-y|,$

then show that first boundary value problem $y''+f(t,y)=0,\ \ y(a)=A, y(b)=B$ has unique solution whenever $\frac{K(b-a)^2}{\pi^2}<1$.

(b) Discuss the failure of existence and uniqueness of the linear boundary value problem

$$y''(t)+y'(t)=0,\quad y(0)=0,\qquad y(b)=B.$$

7. (a) Obtain the existence and unique solution of first boundary value problem

$$y''(t)+f(t,y(t),y'(t))=0,\ y(a)=A, y(b)=B$$

by matching the solutions of two initial value problems, provided $f(t,y,y')$ is continuous and satisfies Lipschitz condition.

(b) Let $f(t,y,y')$ be continuous on $[a,b]\times(-\infty,\infty)\times(-\infty,\infty)$ and satisfies

$$|f(t,y,y')-f(t,x,x')|\le K|y-x|+L|y'-x'|.$$

Then show that the boundary value problem $y''(t)+f(t,y(t),y'(t))=0,$ $y(a)=A, y(b)=B$ has one and only one solution provided

$$\frac{K(b-a)^2}{8}+\frac{L(b-a)}{2}<1.$$

8. (a) Define generalize Lipschitz condition.

(b) Suppose $f(t,y,y')$ be continuous on $[a,b]\times(-\infty,\infty)\times(-\infty,\infty)$ and satisfies

$$|f(t,y,y')-f(t,x,x')|\le p(t)|y-x|+q(t)|y'-x'|.$$

If the equation $u''(t)+q(t)u'(t)+p(t)u(t)=0$ has a solution satisfying $u(t)=0$, $u'(t)=m$ on [a,b]. Then show that the second boundary value problem $y''(t)+f(t,y(t),y'(t))=0,\ y(a)=A, y'(b)=m$ has one and only one solution.

Code: 15 MAT 302

FLUID MECHANICS
SYLLABUS

Unit-1: Basics in Fluid Mechanics:

The continuum hypothesis-Newtonian and Non-Newtonian fluids-Continuity equation-Navier-Stokes equations of motion –Energy equation, steady and unsteady flows.

Unit-2: Navier-Stokes equations:

Parallel flow through a straight channel and Couette flow-The Hagen-Poiseuille flow-The suddenly accelerated plane wall-Stokes first problem. The flow near an oscillating flat plate-Stokes second problem- Flow near a rotating disk. Parallel flow past a sphere.

Unit-3: Boundary Layer Theory:

Derivation of Boundary Layer equations for two dimensional flow-The separation of a Boundary Layer, Skin friction-The Boundary Layer on a flat plate.

Unit-4: Thermal Boundary Layers in Laminar flow:

Exact solution for the problem of temperature distribution in a viscous flow: (i)couette fiow, (ii)Poiseuille flow through a channel with flat walls. Forced and natural flows-Thermal Boundary Layer in forced flow-Parallel flow past a flat plate at zero incidence. Thermal Boundary Layers in natural flow (free convection).

Text Books:

1) Boundary Layer Theory- Dr.Herman Schlicting, Mc.GrawHill Book Company

2) Fluid Mechanics and Fluid Machines-S.K Som & G. Biswas

Reference Books:

1) Textbook of fluid dynamics- F. Chorlton, Van Nostrand, 1963

FLUID MECHANICS
MODEL PAPER

TIME: 3Hrs **Marks: 100**

Answer any five from the following **5X20=100**

1) Write short notes on any four of the following:

Newtonian and Non-Newtonian fluids

Laminar flow and Turbulent flow

Separation of Boundary layer.

Governing equations of a homogeneous viscous(fluid) flow.

Free convection and forced convection .

Compressible & incompressible flows.

2. Obtain the velocity distribution for the parallel flow of a homogeneous incompressible fluid through a straight channel, when

a) Both the walls are at rest and

b) One of the walls is at rest and the other moving in its plane with a constant velocity.

3. Write the governing equation of the flow of a homogeneous incompressible fluid through a pipe of circular cross section with rotational symmetry and obtain expressions for maximum velocity, the mean velocity and the volume rate of flow.

4. Obtain the velocity distribution for the non-steady flow of a homogeneous incompressible fluid near a suddenly accelerated flat plate that moves in its plane with a constant velocity.

5. Discuss the creeping motion of the parallel flow past a sphere and obtain the stokes' equation for the drag.

6. Discuss briefly the boundary layer along a flat plate.

7. Obtain the temperature distribution for the Poiseuille flow through a channel with flat walls when the walls have equal temperatures.

8. Discuss the natural convective boundary layer flow near a hot vertical plate.

Code: 15 MAT 306

SEMI GROUPS

Syllabus

Unit-I : Functions on a semigroup

Semigroup, special subsets of a semigroup, special elements of a semigroup, relation and functions on a semigroup, Transformations, Free semigroups.

Unit-II : Ideals and Related concepts

Subdirect products, Completing prime ideals and Filters, Completely semiprime ideals, Semilattices of simple semigroups, Weekly commutative semigroups, separative semigroups, □ - semigroups.

Unit-III : Ideal Extensions

Extensions and Translations, Extensions of a Weekly Reductive semigroup, strict and pure extensions, Retract Extensions, Dense extensions, Extensions of an Arbitrary semigroups, Semilattice compositions.

Unit-IV : Completely Regular semigroups

Completely regular, completely simple semigroups , semilattices of Rectangular groups, strong semilattice of completely simple semigroups, subdirect product of a semilattice and a completely simple semigroup.

Unit- V : Inverse Semigroups

The natural partial order of an inverse semigroup, partial right congruences on an inverse semigroup, Representations by one-to-one partial transformations, Homomorphisms of inverse semigroups, semilattices of inverse semigroups.

Note : 1. 8 Questions to be set out of which 5 Questions to be answered.

2. Questions should be uniformly distributed from all the units.

Prescribed text Book :

1. Introduction to Semigroups by **Mario Petrich; Charles E. Merrill** Publishing Company.

2. The algebraic theory of semigroups volume II, **By A.H.Clifford and G.B.Preston**

American mathematical society.

Reference Text Book :

1. The Algebraic Theory of Semigroups by **A.H.Clifford and G.B.Preston;** American Mathematical Society, First edition.

SEMIGROUPS

MODEL PAPER

Time: 3 hour **Max Marks:100**

Note: Answer <u>ANY FIVE</u> from the following.

1) For any element *a* of a semigroup S, show that i) $L(a) = a \cup Sa$ ii) $R(a) = a \cup aS$

iii) $J(a) = a \cup aS \cup Sa$

2) If φ is a homomorphism of a semigroup S into a semigroup T, then the relation ρ on S defined by $a \rho b$ ifand only if $a\varphi = b\varphi$, is a congruence on S , and $S / \rho \cong S\varphi$. Conversely , if ρ is a congruence on S ,then the mapping $a \rightarrow a\rho$ is a homomorphism of S onto S/ρ.

3) Show that every semigroup is a subdirect produc t of subdirectly irreducible semigroups.

4) Let S be a semigroup , **I** be a semiprime idel and M be an m-system of S such that $I \cap M = \varphi$ and let M* be any m-system of S maximal relative to the properties : $M \subseteq M^*$, $I \cap M^* =$ □ Then show that $S \setminus M^*$ is a minimal prime ideal of s containing **I**

5) A semigroup S is a retract of every extension if and only if S has an identity.

6) Show that the following conditions on a semigroup S are equivalent.

i) S is completely simple
ii) S is completely regular and simple
iii) S is regular and all its idempotents are primitive.
iv) S is regular and weakly cancellative.
v) S is regular and for any $a, x \in S$, $a = axa$ implies $x = xax$

7) If H be an inverse subsemigroup of the inverse semigroup S. Then show that HW is a closed inverse subsemigroup of S.

8) Show that an effective representation of an inverse semigroup S is the sum of a uniquely determined family of transitive effective representations of S.

Code: 15 MAT 204

NUMBER THEORY
SYLLABUS

Unit 1: Divisibility

Early Number Theory, The Division Algorithm, The Greatest Common Divisor, The Euclidean Algorithm.

Unit II: Congruences

Basic Properties of Congruence, Binary and Decimal Representations of Integers, Linear Congruences and the Chinese Remainder Theorem.

Unit III: Fermat's Theorem

Fermat's Little Theorem and Pseudoprimes, Wilson's Theorem.

UNIT IV: Euler's Generalization of Fermat's Theorem

Euler's Phi-Function, Euler's Theorem, Some Properties of the Phi-Function

Unit V: Quadratic Reciprocity Law

Euler's Criterion, The Legendre Symbol and Its Properties, Quadratic Reciprocity, Quadratic Congruences with Composite Moduli.

Text Book: David M. Burton, Elementary Number Theory, Sixth Edition, McGrawHill.

NUMBER THEORY

MODEL PAPER

Time: 3 hour **Max Marks:100**

Note: Answer <u>ANY FIVE</u> from the following.

1 (a) If n is an odd integer, show that $n^4 + 4n^2 + 11$ is of the form 16k.

(b) Let a and b be integers, not both zero. Then prove that a and b are relatively prime if and only if there exist integers x and y such that 1 = ax + by.

2 (a) If ca $\equiv$ cb (mod n), then prove that a$\equiv$ b (mod n/d), where d $\equiv$ gcd(c, n).

(b) Use the theory of congruences to verify that $89/l2^{44}-1$ and $97/2^{48}-1$

3 (a) The linear congruence ax $\equiv$ b (mod n) has a solution if and only if d / b, where d = gcd(a, n). If d / b, then it has d mutually incongruent solutions modulo n.

(b) State and prove Chinese remainder theorem.

4 (a) State and prove Fermat's theorem.

(b) Let n be a composite square-free integer, say, n = p1p2 • • • p_r, where the Pi are distinct primes. If Pi - 1 / n - 1 for i = 1, 2, ... , r, then n is an absolute pseudoprime.

5 (a) State and prove Wilson's theorem.

(b) Using Wilson's theorem, prove that for any odd prime p, $1^2 . 3^2 . 5^2 ... (p - 2)^2 \equiv (-1)^{(p+l)/2}$ (mod p)

6 (a) Prove that the function ϕ is a multiplicative function.

(b) Prove that for $n \geq 2$, ϕ (n) is an even integer.

7 (a) State and prove law of quadratic reciprocity.

(b) Find odd primes $p \neq 3$ for which 3 is a quadratic residue using quadratic reciprocity law.

8 (a) Let p be an odd prime and gcd(a , p) = 1. Then prove that a is a quadratic residue of p if and only if a$(^{p-l)/2} \equiv$ (mod p).

(b) Solve the quadratic congruence $x^2 \equiv 7 \pmod{3^3}$.

Code: 15 MAT 309

CRYPTOGRAPHY

SYLLABUS

Unit I: Introduction

Encryption schemes, symmetric and asymmetric cryptosystems, cryptanalysis, alphabets and words, permutations, block ciphers and stream ciphers.

Unit II: Perfect Secrecy

Perfect Secrecy, Birthday Paradox, Vernam One Time Pad, Random Numbers, Pseudorandom Numbers.

UNIT III: Public Key Cryptography

Principle of Public Key Cryptography, RSA Cryptosystem, Cryptanalysis of RSA, Diffie-Hellman (DH) Key Exchange Protocol, Discrete Logarithm Problem (DLP), ElGamal Cryptosystem.

UNIT IV: Cryptographic Hash Functions

Hash and Compression Functions, Security of Hash Functions, SHA-1, Others Hash Functions, Message Authentication Codes.

Unit V: Digital Signatures

Security Requirements for Signature Schemes, RSA Signature, ElGamal Signature, Digital Signature Algorithm (DSA), Undeniable Signature, Blind Signature.

Text Book: J. Buchmann, Introduction to Cryptography, Springer (India) 2004

CRYPTOGRAPHY

MODEL PAPER

Time: 3 hour **Max Marks:100**

Note: Answer <u>ANY FIVE</u> from the following.

1 (a) What do you mean by Cryptography? Explain the different types of ciphers.

(b) Explain the use of block cipher in ECB mode.

2 (a) Describe the concept of birthday paradox.

(b) State and prove Shannon theorem of perfect secrecy.

3 (a) Explain RSA public cryptosystem.

(b) Define the Diffe Hellman (DH) secret key exchange (SKE) protocol

4 (a) What are hash functions? Explain birthday attack on hash functions.

(b) Describe the hash function SHA-1

5 (a) Explain Vernam one-time pad cryptosystem?

(b) Describe the use of random number generator in Vernam one-time pad.

6 (a) What is an arithmetic compression function? Give an example.

(b) Explain message authentication codes.

7 (a) Explain the idea of digital signatures.

(b) Explain the security and efficiency of Digital Signature Algorithm.

8 (a) Describe the Chaum blind signature protocol.

(b) Explain attacks against RSA digital signature.

Code: 15 MAT 205

SPECIAL FUNCTIONS

SYLLABUS

UNIT-I: The Gamma and Beta Functions

The Gamma function ,A series for $\Gamma'(z)/\Gamma(z)$,Evaluation of $\Gamma'(1)$, the Euler product for $\Gamma(z)$,the difference equation $\Gamma(z+1) = z\Gamma(z)$,evaluation of certain infinite products , Euler 's integral for $\Gamma(z)$, the Beta function , the value of $\Gamma(z)\,\Gamma(1-z)$, the factorial function , Legendre 's duplication formula , Gauss multiplication theorem , a summation formula due to Euler .

UNIT-II: BESSEL FUNCTIONS

Definition of J_n (x) , Bessel's differential equation , Differential recurrence relation, A pure recurrence relation, A generating function, Bessel's integral, Index half an odd integral, modified Bessel function, orthogonality property for J_n (x).

UNIT-III: LEGENDRE'S POLYNOMIALS

Definition of P_n (x) , Differential recurrence relations, the pure recurrence relation, Legendre's differential equation, the Rodrigue's formula , orthogonality property , special properties of P_n (x), more generating functions, Laplace's first Integral form , Expansion of x^n

UNIT-IV: HERMITE POLYNOMIALS

Definition of H_n (x),Recurrence relations , the Rodrigue's formula ,other generating functions ,integrals, the Hermite polynomials as ${}_2F_0$, orthogonality , expansion of polynomial s, more generating functions.

UNIT-V: LAGUERRE POLYNOMIALS

The Laguerre polynomial definition , generating functions, , recurrence relations, the Rodrigue's formula, the differential equation , orthogonality, expansion of polynomials , special properties ,other generating functions , the simple Laguerre polynomials.

TEXT BOOK:

(1) Special functions by E.D. Rainville, MacMillan company, New York, 1960.

SPECIAL FUNCTIONS

MODEL PAPER

Time:3 hours **Max Marks:100**

Note: Answer ANY FIVE from the following.

1 (a) Find the relation between the beta and the gamma function.

(b) Evaluate $\int_0^{\pi/2} \sqrt{\tan\theta}\, d\theta$.

2 (a) State and prove the Legendre's duplication formula.

(b) Evaluate $\int_0^{\infty} e^{-ax} x^{m-1} \sin bx\, dx$ **, by using gamma function.**

3 (a) Prove the orthogonality property for the Bessel function.

(b) Show that $J_n'(x) = \frac{1}{2}[J_{n-1}(x) - J_{n+1}(x)]$.

4 (a) State and prove the Rodrigue's formula for Legendre polynomials.

(b) Express the polynomial $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ **in terms of Legendre polynomials.**

5 (a) State and prove the generating function for Hermite polynomials.

(b) Prove that $2xH_n(x) = 2nH_{n-1}(x) + H_{n+1}(x)$.

6 (a) Show that $\int_0^{\infty} e^{-x} L_m(x) L_n(x) dx = 0,\ m \neq n$.

(b) Evaluate $\int_0^{\infty} e^{-2x}\, [L_3(2x)]^2 dx$.

7 (a) Using Rodrigue's formula, show that $P_n(x)$ **satisfies the differential equation**

$$\frac{d}{dx}\left[(1+x)^2 \frac{d}{dx}[P_n(x)]\right] + n(n+1)P_n(x) = 0.$$

(b) Prove that $\int_0^{\infty} e^{-ax} J_0(bx) dx = \frac{1}{\sqrt{a^2+b^2}}$.

8 (a) Evaluate $\int_{-\infty}^{\infty} e^{-x^2}\, [H_2(x)]^2 dx$.

(b) Prove that $L_n'(x) = L_{n-1}'(x) - L_{n-1}(x)$.

Code: 15 MAT 310

HYPER GEOMETRIC FUNCTIONS AND LIE –GROUPS

SYLLABUS

UNIT-I: THE HYPERGEOMETRIC FUNCTION

The function F (a, b; c; z), A simple integral form, F(a,b;c;1) as a function of the parameters, Evaluation of F(a,b;c;1), The hypergeometric differential equation, F(a, b; c; z) as of its parameters, Elementary series manipulations, Simple transformations, Relation between functions of z and 1-z, A quadratic transformation, Additional properties.

UNIT-II: GENERALIZED HYPERGEOMETRIC FUNCTIONS

The function pFq ,The exponential and binomial functions, A differential equation , other solutions of the Differential equation , A Simple integral , The pFq with unit argument, Saalschutz's Theorem, Whippele's Theorem, Dixon's Theorem, A useful integral.

UNIT-III: THE CONFLUENT HYPERGEOMETRIC FUNCTION

Basic properties of the ${}_1F_1$, Kummer's first and second formula, A theorem due to Kummer.

Generating functions : The generating function concept, generating functions of the form $G(2xt – t^2)$, sets generated by $e^t\,\Phi(xt)$, the generating functions A(t)exp(-xt/1-t).

UNIT-IV: LIE ALGEBRAIC TECHNIQUE

Lie groups, Lie algebras and one parameter subgroups, homomorphism, linear differential operators, Preliminary observations, The Laguerre function, $L_n^{(\alpha)}(x)$, the hypergeometric function ${}_2f_1$ (-n, α; β; x), the modified Laguarre function $L_n^{(\alpha-n)}(x)$.

UNIT-V: THE WEISNER METHOD

Introduction, The differential equation, linear differential operators, group of operators, the extended form of the group generated by B and C, Generating functions for modified Laguerre polynomials, Simple Bessel functions, Gegenbauer polynomials

TEXT BOOK:

(1) Special functions by E.D. Rainville, MacMillan Company, New York, 1960.
(2) A treatise on generating functions by H.M.Srivastva and H.L.Manocha, Halsted/Wiley New York, 1984.
(3) Obtaining Generating functions by Mc.Bride, springer verlag, New York, 1971.

HYPERGEOMETRIC FUNCTIONS AND LIE - GROUPS

MODEL PAPER

Time: 3 hour **Max Marks:100**

Note: Answer ANY FIVE from the following.

1. **Derive the differential equation for the hypergeometric function $_2F_1$ (a , b ; c ; z) and express the exponential and the binomial function in terms of hypergeometric function.**
2. **State and prove the Dixon's theorem.**
3. **Define the confluent hypergeometric function and derive the Kummer's first and second formula.**
4. **Explain Lie algebraic (Special linear group SL(2, C)) technique to obtain the**

   generating function.
5. **Define a generating function and explain various types of generating functions.**
6. **Prove that** $(\square - \square)^{-\square-\square}\square\square\square\left(\frac{-\square\square}{\square-\square}\right) = \sum_{\square=\square}^{\infty} \square_{\square}^{(\square)}(\square)\square^{\square}$.
7. **Derive the recurrence relations of ascending and descending type for $_2F_1$ (-n ,a ;b ; x).**
8. **Write in brief the Weisener's method of deriving generating functions.**

Code: 15 MAT 301

FLUID DYNAMICS
SYLLABUS

Unit I

Kinematics of Fluids in motion: Real fluids and Ideal fluids- Velocity of a fluid at a point, Stream lines , path lines , steady and unsteady flows- Velocity potential - The vorticity vector- Local and particle rates of changes - Equations of continuity - Worked examples - Acceleration of a fluid – Conditions at a rigid boundary.

Unit II

Equations of motion of a fluid: Pressure at a point in a fluid at rest - Pressure at a point in a moving fluid - Conditions at a boundary of two inviscid immiscible fluids- Euler's equation of motion - Discussion of the case of steady motion under conservative body forces.

Unit III

Some three dimensional flows: Introduction- Sources, ranks and doublets - Images in a rigid infinite plane - Axis symmetric flows – Stokes stream function.

Unite IV

Some two dimensional flows: Meaning of two dimensional flow - Use of Cylindrical polar coordinate - The stream function - The complex potential for two dimensional , irrotational in compressible flow - Complex velocity potentials for standard two dimensional flows - Some worked examples – Two dimensional Image systems - The Milne Thompson circle Theorem.

Unit V

Viscous flows: Stress components in a real fluid. - Relations between Cartesian components of stress- Translational motion of fluid element - The rate of strain quadric and principle stresses - Some further properties of the rate of strain quadric - Stress analysis in fluid motion - Relation between stress and rate of strain- The coefficient of viscosity and Laminar flow - The Navier – Stokes equations of motion of a Viscous fluid.

Contents : F. Chorlton, Text Book of Fluid Dynamics ,CBS Publications. Delhi,1985.
Unit 1: Chapter 2. Sec 2.1 to 2.10.
Unit 2: Chapter 3. Sec 3.1 to 3.7.
Unit 3: Chapter 4. Sec 4.1 to 4.5.
Unit 4: Chapter 5. Sec 5.1 to 5.8.
Unit 5: Chapter 8 Sec 8.1, t0 8.9.

REFERENCE(S)

[1] G.K. Batchelor, An Introduction to Fluid Mechanics, Foundation Books, New Delhi, 1984.
[2] A.J. Chorin and A. Marsden, A Mathematical Introduction to Fluid Dynamics, Springer-Verlag, New York, 1993.
[3] S.W. Yuan, Foundations of Fluid Mechanics, Prentice Hall of India Pvt Limited, New Delhi, 1976.
[4] R.K. Rathy, An Introduction to Fluid Dynamics, Oxford and IBH Publishing Company, New Delhi, 1976.

FLUID DYNAMICS
MODEL PAPER

TIME: 3Hrs

Answer any five from the following **5X20=100**

1. (a) Explain the Eulerian and Lagrangian method of describing fluid motion.

(b) Define stream lines, path lines and streak lines. Determine the equation of stream lines, if $\bar{q} = xI - yJ$.

2. (a) Derive the relation between Stress and Rate of strain for an incompressible fluid.

(b) For a fluid moving in a fine tube of variable section A, Prove that the first principles the equation of continuity is $A\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial s}(A\rho v) = 0$

where v is the speed at a point P of the fluid and s the length of the tube up to P. what does this become for steady incompressible flow?

3. (a) Prove that the equation of motion of a homogeneous inviscid liquid moving under forces arising from potential V is

$$\frac{\partial q}{\partial t} - q \wedge \zeta = -\text{grad}\left(\frac{p}{\rho} + \frac{1}{2}q^2 + \nabla\right) \text{where } \zeta = \text{curl}\, q \,\text{is the vorticity.}$$

(b) A two dimensional incompressible steady flow field with velocity components in the rectangular coordinates given by

$u(x,y) = \frac{k(x^2 - y^2)}{(x^3 + y^3)}, \quad v(x,y) = \frac{2xy}{(x^3 + y^3)}$ where k is an arbitrary constant non-zero constant. Verify whether the continuity is satisfied?

4. (a) Derive the equation of the continuity in the vector form.

(b) If $\int A.ds = 0$ for all closed curves in a region R show that there exists a scalar function V such that $A = -\text{grad}\, V$.

5. (a) Define the vorticity ω in the motion of a continuous medium with velocity $v(x,y,z,t)$. Show that for a motion an inviscid incompressible fluid of uniform density, under gravity, the vorticity satisfies the equation $\frac{\partial \omega}{\partial t} + (v.\nabla)\omega = (\omega.\nabla)v$ and explain the significance of each term in this equation.

6. (a) Derive Navier-Stokes equation for the flow of an incompressibe fluid in vector form.

(b) Consider an incompressible steady flow with constant velocity. The velocity components are given by

$$u(y) = y\frac{U}{h} + \frac{h^2}{2\mu}(-\frac{dp}{dx})\frac{y}{h}(1 - \frac{y}{h})$$

$$v = w = 0$$

If the body force is neglected, does $u(y)$ satisfy the equation of motion? $h, U \text{ and } \frac{dp}{dx}$ are constants, and $p = p(x)$.

7. (a) Define Stress at a point. Verify the equality of the shearing stresses $\sigma_{yz} = \sigma_{zy}$.

(b) Let the new coordinate system (x', y') be obtained from the original coordinate
system (x, y) by a rotation through an angle 45^0. Verify the invariants of the rates strain for a rectilinear flow with a linear velocity profile, i.e., u=ay, v=0.

8 (a) Derive the boundary layer equations in two dimensional flow.

(b) Derive Blasius Solution of an incompressible fluid past a thin flat plate.

Code: 15 MAT 304

FUZZY ALGEBRA

SYLLABUS

Unit-1 Fuzzy subsets & Fuzzy sub groups

Union of two fuzzy subgroups, fuzzy subgroup generated by a fuzzy subset, fuzzy normal subgroups, fuzzy conjugate subgroups and fuzzy characteristic subgroups, fuzzy syllow subgroups.

Unit-2 Fuzzy sub rings and Fuzzy ideals

Basic concepts, properties of fuzzy ideals, union of fuzzy sub rings (fuzzy ideals), fuzzy sub ring (fuzzy ideal) generated by a fuzzy subset, fuzzy ideals and homomorphism, fuzzy cosets.

Unit-3 Fuzzy prime ideal and Maximal ideals

Fuzzy prime ideals, fuzzy maximal ideals, fuzzy semi prime ideals, characterization of regularity.

Unit-4 Fuzzy primary ideals

Fuzzy primary ideals, fuzzy semi primary ideals definition and some properties, fuzzy ideals and irreducible ideals in Noetherian ring.

Note : 1. 8 Questions to be set out of which 5 Questions to be answered.

2. Questions should be uniformly distributed from all the units.

Prescribed Text Book:

Fuzzy Algebra by **Rajesh Kumar** ; University Press, University of Delhi, Delhi-110007.

Reference Text Book:

Fuzzy Commutative Algebra by **John N Mordeson & D S Malik;** World Scientific Publishing Co. Pte. Ltd.

FUZZY ALGEBRA

MODEL PAPER

Attempt any five questions from the following **5x20 = 100M**

1. A fuzzy subset μ of a group G is a fuzzy subgroup of G iff, the level subsets μ_t, t ϵ Im μ, are subgroups of G.

2. For a fuzzy subgroup μ of G, the following statements are equivalent

(i) μ is a fuzzy characteristic subgroup of G.

(ii) Each level subgroup of μ is a characteristic subgroup of G.

3. If $\{\mu_n \mid n \in Z^+\}$ is a collection of fuzzy ideals of a ring R such that $\mu_1 \subseteq \mu_2 \subseteq \mu_3 \subseteq \ldots\ldots \subseteq \mu_n \subseteq \ldots$, then $\cup \mu_n, n \in Z^+$ is a fuzzy ideal of R.

4. Let f be a homomorphism from a ring R onto a ring R´ and let μ be any f - Invariant fuzzy ideal of R, then Rμ ≅ R´f(μ).

5. If f is a homomorphism from a ring R onto a ring R´ and μ´ is any fuzzy prime ideal of R´, then $f^{-1}(\mu')$ is a fuzzy prime ideal of R.

6. If μ is any fuzzy prime ideal of a ring R, then $(\sqrt{\mu^2}) = \mu$, where $\mu^2 = \mu \circ \mu$.

7. If μ is any fuzzy primary ideal of a ring R then μ_t, t ϵ Im μ, is a primary ideal of R.

8. If μ is any fuzzy primary ideal of a ring R, then the ring Rμ is primary.

Code: 15 MAT 313

DIFFERENTIAL GEOMETRY

SYLLABUS

UNIT-I:

Curves in the plane in space: Curve-Arc-Length- parameterization-Level Curves vs. parameterized curves- Curvature – Plane curves – Space Curves

UNIT-II:

Global properties of curves: Simple closed curves-The Isoperimetric Inequality – The Four Vertex Theorem-Surfaces in Three Dimensions: Surface-Smooth Surfaces-Tangents, Normals and orientability – Examples of Surfaces – Quadratic

UNIT-III:

The First fundamental form: Lengths of Curves on surfaces – Isometric of Surfaces – Conformal Mappings of Surfaces – Surface Area – Equi-areal Maps and a Theorem of Archimedes-Curvature of Surfaces - The second Fundamental form–The Curvature of Curves on a Surface.

UNIT-IV:

Topological spaces: Definitions and examples–Elementary concepts-Open bases and open sub bases – Weak topologies.

UNIT-V:

Compactness: Compact Spaces -Product Spaces–Tychnoff's theorem.

Text Books:

1. Elementary Differential Geometry by Andrew Pressley, Springer.
2. Three Dimensional Differential Geometry by Bansilal.
3. Introduction to Topology and Modern Analysis by G.F. Simmons.
4. General Topology by Schaum series.

DIFFERENTIAL GEOMETRY
Model Paper

1. Find the parameterizations of the following level curves.

i) $y^2 - x^2 = 1$ ii) $\frac{x^2}{4} + \frac{y^2}{9} = 1$

2. Find the Cartesian equations of the following parameterized curves and also find their tangent vectors

i) $\gamma(t) = (\cos^2 t, \sin^2 t)$ ii) $\gamma(t) = (e^t, t^2)$

3. Show that the following curves are of unit speed.

i) $\gamma(t) = \left(\frac{1}{3}(1+t)^{\frac{3}{2}}, \frac{1}{3}(1-t)^{\frac{3}{2}}, \frac{t}{\sqrt{2}}\right)$ ii) $\gamma(t) = \left(\frac{4}{5}\cos t, 1-\sin t, \frac{3}{5}\cos t\right)$

4(a). Find the curvature of the circular helix $\gamma(\theta) = (a\cos\theta, a\sin\theta, b\theta)$ $, -\infty < \theta < \infty$

(b). Compute the torsion of the circular helix $\gamma(\theta) = (a\cos\theta, a\sin\theta, b\theta)$ $, -\infty < \theta < \infty$

5. Show that the ellipse $\gamma(t) = (a\cos t, b\sin t)$, where a and b are positive constants, is a simple closed curve.

6. State and prove the four vertex theorem.

7(a). Show that the unit sphere $x^2 + y^2 + z^2 = 1$ is of smooth surface.

(b). Find the tangent plane of the surface $\sigma_{u,v} = (u, v, u^2 - v^2)$ at (1, 1, 0).

8(a). Prove or disprove that any closed subspace of a compact space is compact.

(b). Prove that any continuous image of a compact space is compact.

Code: 15 MAT 314

INVENTORY MODELS

SYLLABUS

UNIT-I

Deterministic inventory Models

Inventory – Types of Inventory – Inventory Decisions – Classification of Inventory Models – Concept of Average inventory – Economic Ordering Quantity (EOQ) – EOQ with shortages and without shortages - EOQ with constraints.

UNIT-II

Dynamic or Fluctuating Demand Models

Dynamic or Fluctuating Demand Models: Re-order level – Optimum Buffer stock – Inventory Control System – Deterministic Models with Price- Breaks.

UNIT-III

Probabilistic Inventory Models

Instantaneous Demand – No Set- up cost Model – Uniform Demand - No Set- up cost Model - Probabilistic order-level system with constant lead Time – Multi –period Probabilistic model with constant lead Time.

UNIT-IV

Selective Inventory Management

ABC Analysis – VED Analysis- XYZ Analysis Based on inventory value – FNSD Analysis Based on Usage rate of items

UNIT-V

Markov Analysis

Introduction-Stochastic Process-Markov Process-Transition Probability-n-Step transition Probabilities-Markov Chain-Chapman-Kolomogrov Theorem

Reference Books:

1. S.D.Sarma,”Operations Research”
2. J.Medhi,”Stochastic Processes”, New Age International
3. Prem kumar Gupta,D.S.Hira,”Operations Research”,S.Chand &Company Ltd.

INVENTORY MODELS

TIME: 3Hrs **Marks: 100**

Answer any five from the following **5X20=100**

1(a) What is an inventory system? Explain clearly the different costs that are involved in inventory problems with suitable examples.

(b) What is Economic Order Quantity? Discuss step by step, the development of EOQ.

2(a) A manufacturer has to supply his customer with 600 units of his product per year. Shortages are not allowed and shortage cost amounts to 60 paise per unit per year. The set up cost per run is Rs. 80.00. Find the following.

(i) The Economic Order Quantity.
(ii) The minimum average yearly cost.
(iii) The optimum number of orders per year.
(iv) The optimum period of supply per optimum order.
(v) The increase in the total cost associated with ordering a) 20% more and b) 40% less than EOQ.

(b) The annual demand for a product is 64000 units (or 1280 units per week). The buying cost per order is Rs. 10 and the estimated cost of carrying one unit in stock for a year is 20%. The normal price of the product is Rs. 10 per unit. However, the supplier offers a quantity discount of 2% on an order of atleast 1000 units at a time, and a discount of 5% if the order is for atleast 5000 units. Suggest the most economic purchase quantity per order. `

3(a) With the help of a Quantity – Cost curve, explain the significance of Economic Order Quantity. Derive the Wilson's EOQ formula.

b) What are the limitations in using the formula for EOQ? Discuss its sensitivity.

4(a) Formulate and solve the purchase inventory problem with one price break.

b) Discuss the working rule to obtain the optimum purchase quantity when two quantity discounts are given.

5(a) Discuss the probabilistic inventory model with instantaneous demand and no step-up cost.

(b) Find the optimal quantity Z in a continuous simple stochastic model for a time dependant case. Shortages are allowed and backlogged fully, set-up cost per period is constant.

6(a) A News paper boy buys papers for Rs. 2.60 each and sells them fro Rs. 3.60 each. He cannot return unsold news papers. Daily demand has the distribution.

No. of customers	23	24	25	26	27	28	29	30	31	32
Probability	0.01	0.03	0.06	0.10	0.20	0.25	0.15	0.10	0.05	0.05

If each day's demand is independent of the previous day's , how many papers should he order each day?

7(a) Explain 'ABC' analysis. What are its advantages and limitations if any?

(b) Perform ABC analysis on the following sample of items in an inventory.

Item name	Annual consumption	Price per unit (in paise)
A	300	10
B	2800	15
C	30	10
D	1100	5
E	40	5
F	220	100
G	1500	5
H	800	5
I	600	15
J	80	10

8(a) Define the terms i) Markov Process, ii) Transition Probability, iii) Stochastic Matrix, iv) Ergodic process, v) Equilibrium of steady state.

(b) A house-wife buys three kinds of cereals viz A, B, C. She never buys the same cereal on successive weeks. If she buys cereal A, then the next week she buys cereal B. However, if she buys B or C , then next week she is three times as likely to buy A as the other brand. Find the transition matrix. In the long run, how often she buys each of these brands.

Code: 15 MAT 308

DIFFERENTIAL EQUATIONS

Eight questions are to be set and the student has to answer five in three hours of duration

UNIT-I : System of linear differential equations: system of first order equations, existence and uniqueness theorem, fundamental matrix, non- homogeneous linear system, linear systems with constant coefficients.

UNIT-II : Existence and Uniqueness of solutions: Introduction, preliminaries, successive approximations, Picard's theorem, continuation and dependence on initial condition, existence of solutions in the large interval.

(Scope and treatment as in Chapters : 4 and 5 of Text book (1))

UNIT- III : Oscillation theory and boundary value problems : Qualitative properties of solutions, the sturm comparison theorem, Eigen values, Eigen functions

UNIT- IV : Power series solutions and special functions : Series solutions of first order and second order linear differential equations, ordinary points, regular singular points. Gauss's hyper geometric equation, the point at infinity.

UNIT V : Non-linear equations : Autonomous systems, the phase plane and its phenomena, type of critical points, stability, critical points and stability for linear systems, stability by Liapunov's direct method, simple critical points of non linear systems.

(Scope and treatment as in Chapters : 4 ,5(sections 25-29) and 8 (sections 40-44)of Text book (2))

Text Books :

1. Text book of ordinary differential equations by S.G.Deo, V. Lakshmikantham and V. Raghavendra, second Edition, Tata McGraw - Hill publishing Company Ltd., New Delhi, 2002.
2. Differential equations with applications and historical Notes by George F. Simmons, Tata McGraw - Hill publishing Company Ltd., New Delhi, 1972.

DIFFERENTIAL EQUATIONS

Time : 3 hours **Max. Marks : 100**

Answer any **Five** questions from following, each question carries equal marks.

1. (a) Find the fundamental system of solutions for the system of equations

$$x_1'(t) = x_1(t), \quad x_2'(t) = 2x_2(t), \text{ for all } t \in [0,1].$$

(b) Compute the solution of the following non-homogeneous system $x' = Ax + b(t)$, where

$$A = \begin{bmatrix} 3 & 2 \\ 0 & 3 \end{bmatrix}, b(t) = \begin{bmatrix} e^t \\ e^{-t} \end{bmatrix} \text{ and } x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

2. (a) Determine exp(At) for the system $x' = Ax$, where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -11 & 6 \end{bmatrix}.$$

(b) Prove that if x(t) is the solution of the initial value problem $x' = f(t,x), x(t_0) = x_0$ on Some interval if and only if x(t) is solution of the corresponding integral equation.

3. (a) If f(t, x) is continuous function on |t| ≤ ∞, |x| < ∞ and satisfies Lipschitz condition on the strip S_a for all a > 0, where Sa = {(t, x) : |t| ≤ a, |x| < ∞}. Then show that the initial value problem $x' = f(t,x), x(t_0) = x_0$ has a unique solution existing for all t.

(b) State and prove Picard's theorem.

4. If $y_1(x)$ and $y_2(x)$ are two linearly independent solutions of

$$y^{11} + p(x)y^1 + q(x)y = 0$$

then the zeros of these functions are distinct and occur alternately in the sense that $y_1(x)$ vanishes exactly once between any two successive zeros of $y_2(x)$, and conversely.

5. (a). Express $\sin^{-1} x$ in the form of a power series $\sum a_n x^n$ by solving $y^1 = (1-x)^{-1/2}$

Use this result to obtain the formula

$$\frac{\pi}{6} = \frac{1}{2} + \frac{1}{2}.\frac{1}{3}.\frac{1}{2^3} + \frac{1.3}{2.4}.\frac{1}{5.2^5} + \frac{1.3.5}{2.4.6}.\frac{1}{7.2^7} + \dots\dots\dots\dots\dots\dots\dots$$

(b). Find the general solution $(1+x^2)y^{11} + 2xy^1 - 2y = 0$ of in terms of power series in x. Can you express this solution by means of elementary functions?

6. (a). Find the power series solution of $x^2 y^{11} + (3x-1)y^1 + y = 0$

(b). Find two independent Frobenius solutions of each of the equations

$$x^2 y^{11} - x^2 y^1 + (x^2 - 2)y = 0$$

7. Describe the relation between the phase portraits of the systems

$$\begin{cases} \dfrac{dx}{dt} = F(x, y) \\ \dfrac{dy}{dt} = G(x, y) \end{cases} \quad \text{and} \quad \begin{cases} \dfrac{dx}{dt} = -F(x, y) \\ \dfrac{dy}{dt} = -G(x, y) \end{cases}$$

8. Sketch the phase portrait of the equations $\dfrac{d^2 x}{dt^2} = 2x^3$, and show that it has an unstable isolated critical point at the origin.

Code: 15 MAT 207

DISTRIBUTIONS & ESTIMATION THEORY

Syllabus

Unit 1 : **DISTRIBUTIONS**

Discrete And Continuous Distributions (Binomial,Poisson,Geometric,Hyper Geometric ,Rectangular,Normal,Gamma Distributions and their Properties),Bi-Variate and Multivariate Normal Distributions, Exponential Family of Distributions

Unit 11: ***LIMIT THEOREMS***

Modes of convergence, Weak law of large numbers, Strong law of large numbers. Limiting moment generating functions, Central limit theorem.

Unit 111: SAMPLE MOMENTS AND THEIR DISTRIBUTIONS

Random sampling, sample characteristics and their distributions- x^2 ,t and F distributions distribution of $(\bar{X}, S^2)$ in sampling from a normal population. Sampling from a Bi-variate normal distribution

Unit IV : ***THEORY OF POINT ESTIMATION***

Problem of point estimation, Properties of estimates, Unbiased estimation, Lower bound for variance of estimate, Rao- Blackwell theorem, Method of moments, Maximum likelihood estimates, Bayes & Minimax estimation, Minimal sufficient statistic

Unit V : ***CONFIDENCE INTERVAL ESTIMATION***

Shortest length confidence intervals, Relation between confidence estimation and hypothesis testing, unbiased confidence intervals, Bayes confidence intervals

PRESCRIBED BOOK

An introduction to Probability theory and Mathematical Statistics-V.K. Rohatgi, Wiley Eastern Publications first edition- 1975) [Chapters 5,6,7,8,11]

Additional Reading: Introduction to Mathematical Statistics (Fourth edition)
Robert Hogg & Allen Craig

DISTRIBUTION AND ESTIMATION THEORY

Time:3Hours **Max.Marks: 100**

Answer any FIVE questions all questions carries equal marks.

1. a) Obtain the mean and variance of a truncated Binomial distribution truncated at X=0.
   b) Derive the p.d.f. of Poisson distribution truncated at the origin and find its mean and variance.
2. a) Let X_1, X_2 be independent random variable with Xi follows b(n_i,$\frac{1}{2}$), i=1,2. What is the PMF for X_1- X_2+ n_2?
   b) Let X and Y be independent geometric RVs. Show that min(X,Y) and X-Yare independent.
3. a) Let $X_n \xrightarrow{p} X$, and g be continuous function defined on R. Then $g(X_n) \xrightarrow{p} g(X)$ as n $\rightarrow\infty$.
   b) State and prove Borel-Cantelli Lemma.

4. a) Let (X_1, Y_1), (X_1, Y_1), . . . , (X_1, Y_1) be a sample from a bi-variate population with variances σ_1^2, σ_2^2 and covariance $\rho\sigma_1\sigma_2$. Then E$ES_1^2 = \sigma\,_1^2$, $ES_2^2 = \sigma\,_2^2$ $and\ ES_{11} = \sigma_1\sigma_2$

b) Let X_1, X_2, . . . , X_n be a random sample from $N(\mu, \sigma^2)$. Compute the first four sample moments of $\bar{X}$ about the origin and abut the mean Also compute the fit four sample moments of S^2about the mean.

5. a) Derive the characteristic function of a chi-square distribution. Establish its reproductive property.

b). Let X and Y be independent normal RVs. A sample of n=11 observations on (X,Y) produces sample correlation coefficient r=.40.Find the probability of obtaining a value of R that exceeds the observed value.

6. a) Find the general form of the distribution of X such that a random sample X_1, X_2, . . . , X_n from the distribution as sufficient statistic.

b) State and prove Rao-Blackwell theorem. State Lehmann and Schaffe theorem. Explain its use.

7. a) Explain moments method of estimation. Under regularity t_y conditions to be stated by you, prove that M.L estimator is asymptotically efficient.

b) Let X_1, X_2, . . . , X_n be a sample from G(1,θ). Find the shortest-length confidence interval for θ at level (1-α), based on a sufficient statistic for θ.

8. a) Let X_1, X_2, . . . , X_n be i.i.d. with PDF $f_\theta(x) = \frac{\theta}{x^2}\ , x \geq \theta, and = 0\ otherise.$ Find the shortest length (1-α)-level unbiased confidence interval for θ based on the pivot $\frac{\theta}{X_{(1)}}$.

b) Let X_1, X_2, . . . , X_n be a sample from $U(0, \theta)$. Show that the unbiased confidence interval for θ based on the pivot matrix$\frac{X_i}{\theta}$, coincides with the shortest length confidence interval based on the same pivot.

Code: 15 MAT 312

SAMPLING THEORY
Syllabus

UNIT I- <u>SIMPLE RANDOM SAMPLING</u>

Simple Random Sampling, Selection of a Simple Random Sample, Definitions and Notations, Properties of the Estimates, Variances of the Estimates, Estimation of the Standard Error from A Sample, Confidence Limits, Random Sampling with Replacement, Estimation of a Ratio, Estimates of a Means over Sub Populations, Estimates of Totals Over Sub Populations, Comparison between domain Means, Validity of the Normal Approximation, Linear Estimators of the Population Mean.

UNIT II- <u>STRATIFIED RANDOM SAMPLING</u>

Description, notation, properties of the estimates, the estimated variance and confidence limits, optimum allocation, relative precision of stratified random and simple random sampling, stratification producing large gains in precision, allocation requiring more than 100 percent, estimation of sample size with continuous data, stratified sampling for proportions, estimation of sample size with proportions

UNIT III- <u>RATIO ESTIMATORS</u>: Methods of estimation, the ratio estimate, approximate variance of the ratio estimate, estimation of a variance from a sample, confidence limits, comparision of the ratio estimate is a best linear unbiased estimator, bias of the ratio estimate ,accuracy of the formulas for the variance and estimated variance, ratio estimates in stratified random sampling, the combined ratio estimate, comparison of the combined and separate estimates, short -cut computation of the estimated variance, optimum allocation with a ratio estimate, unbiased ratio type estimates, comparison of the methods, improved estimation of the variance, comparison of two ratios, multivariate ratio estimates, product estimators

UNIT IV- <u>REGRESSION ESTIMATORS</u>:The linear regression estimate , regression estimates with pre-assigned b, regression estimates when b is computed from the sample, sample estimate of variance. Large sample comparision with the ratio estimate and the mean per unit , bias of the linear regression estimate, the linear regression estimate under a linear regression model, regression estimates in stratified sampling, regression coefficients estimated from the sample, comparison of the two types of regression estimates

UNIT V- <u>SYSTERMATIC SAMPLING :</u>Description, relation to the cluster sampling, variance of the estimated mean, comparison of systematic with stratified random sampling, populations in “Random”order, populations with linear trend, methods for populations with linear trends, populations with periodic variation, auto-corelated populations, natural populations, estimation of the variance from a single sample, stratified systematic sampling, systematic sampling in two dimensions, summary

<u>PRESCRIBED BOOK</u>

SAMPLING TECHNIQUES by W.G. COCHRAN, Wiley, Third edition (CHAPTERS: 2,5,6,7,8)

<u>Additional reading</u>

Sampling theory by DES RAJ , McGraw Hill

SAMPLING THEORY

Answer any **five** questions. Each carry 20 marks

1. a) Define Simple random sampling. Explain with illustrations the need for sampling in contrast to census.

   b) Give an estimate of the population mean and its variance in the case of simple random sampling.

2. a) Describe the problems of optimum allocation in stratified sampling and discuss briefly merits and demerits of stratified sampling.

   b) Obtain an unbiased estimate of the population mean and compare its efficiency with that of a simple random sampling.

3. a) Explain stratified sampling with continuous data and sampling for its proportions.

   b) Explain gain in precision in stratified sampling for proportions.

4. a) What is meant by ratio estimator? State the assumptions of ratio estimator.

   b) Derive approximate expressions for bias and MSE of the ratio estimators of the population total assuming SRSWOR for the units.

5. a) Show that under certain conditions to be stated, separate ration estimator is better than combined ration estimator.

   b) Discuss the relative efficiency of ratio and regression estimates.

6. a) Define regression estimators and give an example of its application. Obtain an estimate of the bias of regression estimator for the population total.

   b) What are the conditions under which the regression estimators is as BLUE?

7. a) Explain systematic sampling procedure with the help of an example.

   b) Find the variance of the sample mean in terms of correlation coefficient between pairs of units that are in the same systematic sample.

8. a) Compare a systematic sampling with random sampling and also discuss when the estimator based on systematic sample is more precise.

b) Obtain an unbiased estimate of the population mean and compare its efficiency with that of a simple random sampling.

Code: 15 MAT 208

KL UNIVERSITY
M.Phil / PRE-Ph.D EXAMINATION
DEPARTMENT OF MATHEMATICS
SYALLABUS
MATHEMATICAL METHODS AND STOCHASTIC PROCESSES

Unit-1:

Numerical Analysis:

Numerical solution of simultaneous Linear equations Gauss reduction- Crout Reduction-gauss Jordan Reduction – inverse Of Matrix-Iterative methods-gauss seidel iteration ,Relaxation, Inherent errors

Numerical solution of Non-linear equations-Regular Falsi -Newton Raphson method Iterative Method of Higher order –Solutin of set of Non-Linera ewquations

Graffe's root squaring technique,Bairstow Iteration-scaling Method

Unit-II

Laplace transforms:

The Laplace Transform ,the Inverse Laplace Transform, application to differential equation and Integral and difference equations

Stochastic process:

Unit:III

Stochastic process, Markov chains

Unit:IV:

Markov processes with discrete state space-Poisson process and it s extensions

Unit-V

Markov process with continuous state space

SCOPE OF T HE SYALLABUS;

For unit -1 ,Introduction to Numerical analysis(2ndEdition)by F.B.hiller Band,Tata Mc Graw Hill Publishing company Ltd.

For Unit-II chapter 1to IV in Theory and problems of laplace transformation by Murray B.Spegel Schaum's outline series,McGraw-Hill book company(1989)

For Unit III,IV&V Stichastic process, Medhi.J.Wiley eastern Limited .

Note:
Two- questions on each units I&II
One - questions on each units III&IV&V
One - questionof short notes type are to be set
Five questions to be answered out off 8 questions.

KL UNIVERSITY
M.Phil / PRE-Ph.D EXAMINATION
DEPARTMENT OF MATHEMATICS
MODEL QUESTION PAPER

PAPER-II MATHEMATICAL METHOD AND STOCHASTIC PROCESSES

Time: 3Hours **Max. Marks: 100**

Answer any Five of the following:

1. (a) Describe the Gauss-Seidal iterative method for solving a system of linear equations.
   (b) Solve the following system equations by Gauss-Jordan reduction method.

$$10x_1 + 7x_2 + 8x_3 + 7x_4 = 32$$
$$7x_1 + 5x_2 + 6x_3 + 5x_4 = 23$$
$$8x_1 + 6x_2 + 10x_3 + 5x_4 = 33$$
$$7x_1 + 5x_2 + 5x_3 + 10x_4 = 31$$

2. (a) Describe Graffe's root-squaring technique to find the roots of polynomial equation.
   (b) Find a root of $x^2 - x - 1 = 0$ by Newton-Raphson method correct up to 4 decimals.
3. Define Laplace Transform and state and prove its properties.
4. (a) Describe solving homogeneous difference equation with constant coefficients
   (b) Solve the difference equation
   $U_n = \frac{1}{2}(U_{n+1} + U_{n-1}), 1 \leq n \leq a - 1$ with initial conditions $U_0 = 1$ and $U_a = 0$
5. (a) Define a Markov chain and discuss the classification of states in Markov chain.
   (b) If $P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}, 0 < a, b < 1$, is the transition probability matrix of a Markov chain, then find its stationary distribution
6. (a) Derive Poisson process stating the postulates.
   (b) If $\{N(t)\}$ is a Poisson Process and$s < t$, find $P\left(N(s) = \frac{k}{N(t)} = n\right)$.
7. (a) Define Wiener processes and derive its differential equations.
   (b) If $\{X(t), 0 < t\}$ is a Wiener process with $X(0) = 0$ and $\mu = 0$, then find $P(X(t) = x)$.
   Find the matrix of the complimentary distribution functions of the waiting times.
8. Write short notes on any three of the following:
   (a) Solution of equations by Cotut's method
   (b) Inverse Laplace transformation
   (c) Classification of stochastic processes
   (d) Time dependent Poisson processes
   (e) Stationary of a Markov chain

Code: 15 MAT 311

KL UNIVERSITY
M.Phil / PRE-Ph.D EXAMINATION
DEPARTMENT OF MATHEMATICS
SYLLABUS
QUEUEING THEORY

Unit-I

Concept of queuing theory, some important Random processes, Definition and Classification of Stochastic processes. Discrete-Time Markov Chains, Continuous-Time Markov chains, Birth-Death processes.

Unit-II

Steady state solutions M/M/1/k, M/M/m/k, M/Er/1, Er/M/1, $M^x/M/1, M/M^y/1$ with FCFs.

Unit-III

Embedded Markov chain technique, Non-Poisson queues:$M/G/S, M^X/G/1, GI/M/I$, and $GI/M/S$.

Unit-IV

Elements of priority, tandem and parallel queues.

Unit-V

Optimal design and control of queues. The N-Policy and the T-Policy.

BOOKS FOR STUDY

1. Queuing Systems
   Volume I by Leonard Kleinrock(for Unit-I,II,&III)
2. Elements of queuing theory by Thomas L.Satty(for Unit-IV)
3. Introduction to Queuing theory by Robert B Cooper(for Unit-V)

BOOKS FOR REFERENCE

1. A first course in Bulk queues by M.L.Chaudhary and J.G. Templeten.
2. Application of queuing theory by G.F.Newell.
3. Probability, Statistics and Queuing theory with Computer Science applications by Arnold O.Allen
4. Queues by D.R.CO
5.

Note:1. One question and each of the units I,IV&V, two questions on each of units II&III, and one question of short answer type to be set.

Note:2. Eight questions to be set out of which 5 questions to be answered.

K L UNIVERSITY
DEPARTMENT OF MATHEMATICS
M.Phil/Pre.Ph.D-PART-1 EXAMINATION
MODEL QUESTION PAPER

QUEUEING THEORY

TIME: 3 HOURS **MAX.MARKS:100**

Answer any FIVE questions
All questions carry equal marks

1. (a) State and prove Chapman-Kolmogorov equations. Bring out their application in Markov chain (MC) theory.
   (b) Derive the probability distribution of a simple birth and death process.

2. (a) Explain the essential features of a queueing system. Derive the steady state solution of M/M/1:k/FCFS System
   (b) Derive the waiting time distribution for the system $M/M^y/1$.

3. (a) Derive the steady state solution of M/M/m:k/FCFS system. Obtain it s performance measures.
   (b) What are bulk queues? Obtain the steady state solution of $M^x/M/1$ and hence show that M/M/1 is particular case of it.

4. (a) Explain M/G/1 system. Derive the Pollaczek-Khintchine formula.
   (b) Derive the waiting time distribution for the system GI/M/S. Hence obtain expected waiting time numbers in the system.

5. (a) show that the steady state arrival point system has geometrical distribution in the case of GI/M/1 model
   (b) Obtain the steady state solution of M^x /G/1.derive its waiting time distribution.

6. Explain tandem and parallel queues with suitable examples.

7. What is meant by ‘control of queue’ discuss N-Policy and T-policy.

8. Write short notes on following;
   a) stationary process
   b) Classification of states
   c) E_k/M/1 system
   d) M/G/S system
   e) priority queues

Code: 15 MAT 303

TRIBOLOGY OF BEARINGS

Unit-I: Lubrication

Definition, Types of lubricants, Viscosity, Types of Viscometers, Effect of temperature on Viscosity, Effect of pressure on Viscosity, Other physical properties of mineral oils.

Unit-II: Basic Equations

Generalized Reynolds equation, Flow and shear stress, Energy equation, Equation of state.

Unit-III: Idealized Hydrodynamic Bearings

Mechanism of pressure development in bearings, Plain slider bearing, Idealized slider bearing with a pivoted shoe, Step (Rayleigh step) bearing, Infinitely long journal bearing, Infinitely short journal bearing.

Unit-IV: Squeeze Film Bearings

Parallel surface bearing, Step bearing, A Circular cylinder near a plane, A Parallel circular plate, A Sphere near a plane, A Sphere in a spherical seat, A Rectangular plate on a plane surface, A Journal bearing.

Unit-V: Elastohydrodynamic Lubrication

Hydrodynamic equation, Elastic deformation, Grubin type solution, Accurate solution, Point contact, Dimensionless parameters, Film thickness equations, Different regimes in EHL contacts.

Reference: Introduction to Tribology of Bearings – B.C.MAJUMDAR, S.CHAND

TRIBOLOGY OF BEARINGS

MODEL PAPER

Max. Marks: 100 **Time: 3 hours**

Note: 1. Answer any FIVE Questions

2. Each Question carries 20 Marks.

1. Discuss about the lubricants and their properties. Also write the industrial applications of lubricants?
(5M+15M)

2. Discuss about the different types of viscometers.
$\left(4 \times 5 = 20M\right)$

3. Derive the Generalized Reynolds equation and write its different cases?
(20M)

4. Define the following from the Generalized Reynolds equation :
   (a) Flow and shear stress.
(7M)
   (b) The energy equation.
(7M)
   (c) The equation of state.
(6M)

5. Derive the following in plane slider Bearing:
   (a) hydrodynamic pressure.
   (b) load carrying capacity.
   (c) Friction force.
(d) Coefficient of fluid friction.
$\left(4 \times 5 = 20M\right)$

6. Using different boundary conditions, Write the bearing performance characteristics of the following:
   (a) an infinitely long journal bearing.
   (10M)
   (b) an infinitely short journal bearing.
   (10M)

7. Write the situations under squeeze film lubrication of ANY TWO of the following:
   (a) A circular cylinder near a plane. (10M)
   (b) A parallel circular plate. (10M)
   (c) A sphere near a plane. (10M)
   (d) A rectangular plate on a plane surface (10M)

8. Write down the Grubin type of solution in Elasto-hydrodynamic line contact problem. (20M)

Code: 15 MAT 203

KL UNIVERSITY: GUNTUR
DEPARTMENT OF MATHEMATICS
Pre Ph.D. Examinations
Timescale calculus

Eight questions are to be set and the student has to answer five in three hours of duration:

UNIT-1

Basic definitions: Jump operators, left and right dense, left and right scattered, Induction principle. Differentiation, Properties Leibnitz rule, examples and applications.

UNIT-2 **Integration**: Regulated function rd-continuous, Existence of pre-antiderivative and antiderivative, Mean value theorem, chain rule, Intermediate vale theorem and L'Hopitals rule.

Unit-3 : **First order linear equations:** Hilger's complex plane, the exponential function, examples of exponential functions, The regressive linear dynamic equations, initial value problems and variation of constants formula..

Unit-4 : **Second order linear equations**: Wronskians, Linear operator, Abel's theorem, Hyperbolic and Trigonometric functions, Method of factoring, reduction of order, Euler-Cauchy equations, variation of parameters formula.

Text Books: Dynamic equations on time scales, an introduction with Applications. Martin Bohner and Allan Peterson, BirKhauser, Boston.

KL UNIVERSITY: GUNTUR

DEPARTMENT OF MATHEMATICS

Pre-Ph.D Degree Examination

MODEL PAPER

Timescale calculus

Time: 3Hours **Max.Marks:100**

Answer any FIVE questions from following, each question carries equal marks.

1. Define ,classify and illustrate the following with examples:
   a) Left and right dense,
   b) Left and right scattered
   c) Graininess function.
2. Assume that $f:T \to R$ is a function and let $f:T \in T^k$. Then prove the following:
   (i) If f is differentiable at t, then f is continuous at t.
   (ii) If f is continuous at t and t is right scattered, then fis differentiable at t with $f^{\Delta}(t) = \dfrac{f(\sigma(t)) - f(t)}{\mu(t)}$.
   (iii) If t is right-dense, then fis differentiable at t iff the limit $\lim\limits_{s\to t} \dfrac{f(t)-f(s)}{t-s}$.exists as a finite number.
   (iv) If f is differentiable at t, then $f(\sigma(t)) = f(t) + \mu(t) f^{\Delta}(t)$.

3. a) Find the first derivative t^2 on an arbitrary time scale.
   b) Define regulated and rd-continuous functions with examples.

4. Assume $g:\Re \to \Re$ is continuous and $g:T \to \Re$ is delta differentiable on T^k, and $f:\Re \to \Re$ is continuously differentiable. Then prove that there exists c in the interval [t,σ(t)] with $(f \circ g)^{\Delta}(t) = f'(g(c))g^{\Delta}(t)$.
5. Suppose $y^{\Delta}(t) = p(t)y$ is regressive and $t_0 \in T$. Then prove that $e_p(t,t_0)$ is a solution of the initial value problem $y^{\Delta}(t) = p(t)y$, $y(t_0)=1$ on time scale T.
6. Apply the variation constants formula, solve the following initial value problems
   (i) $y^{\Delta} = 2y + t, \quad y(0) = 0, \quad where \ \ T = \Re$
   (ii) $y^{\Delta} = 2y + 3, \quad y(0) = 0, \quad where \ \ T = Z$
7. Solve the dynamic equation $y^{\Delta\Delta} - (t+3)y^{\Delta} + 3ty = 0$ on the time scale T=N.
8. Determine the solution of Euler-Cauchy dynamic equation $t\sigma(t)y^{\Delta\Delta} - 4ty^{\Delta} + 6y = 0$ on a general time scale $T \subset (0,\infty)$.

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Code: 15 MAT 307

KL UNIVERSITY: GUNTUR
DEPARTMENT OF MATHEMATICS
Pre Ph.D. Examinations
Dynamical systems on Time scales

Eight questions are to be set and the student has to answer five in three hours of duration:

UNIT-1
Self-Adjoint equations:
Wronskian matrix, Lagrange Identity, Abel's formula, Hermitian, Riccati equation. Sturm's separation and Comparison theorems.

UNIT-2
Linear Systems and Higher order equations:
Regressive matrices, Existence and Uniqueness theorem, matrix exponential function, Variation of constants, Liouville's Formula, Constant coefficients.

UNIT-3:
Asymptotic behavior of solutions:
Growth and dichotomy conditions, Levinson's perturbation Lemma properties and applications.

UNIT-4:
Dynamic Inequalities:
Grownwall's Inequality, Bernoulli's Inequality, Holders and Minkowski's inequalities.Lyapunov inequalities.

Text Book:
Dynamic equations on time scales, an introduction with Applications. Martin Bohner and Allan Peterson, BirKhauser, Boston.

KL UNIVERSITY: GUNTUR
DEPARTMENT OF MATHEMATICS
Pre-Ph.D Degree Examination
MODEL QUESTION PAPER
Dynamical systems on Time scales

Time: 3 Hours **Max.Marks:100**

Answer any FIVE questions from following, each question carries equal marks.

1. State and prove the comparison theorem for initial value problems.
2. Let A $\in \Re$ be an nxn matrix valued function on T and suppose that f: $T \to \Re^n$ is rd-continuous. Let $t_0 \in T$ and $y_0 \in \Re^n$. Then prove that the initial value problem $y^\Delta(t) = A(t)y + f(t), y(t_0) = y_0$ has a unique solution y: $T \to \Re^n$.
3. Derive Gronwall's Inequality.
4. Assume $f \in Crd$ and x(t,s) be the Cauchy function for L(x(t)= $(px^\Delta)^\Delta(t) + q(t)x^{\sigma(t)} = 0$ for all t $\in T$ then x(t)= $\int_a^t x(t,s) f(s)\Delta s$ is the solution of the initial value problem Lx=f(t), x(a)=0, $x^\Delta(a) = 0$.
5. Write the dynamic equation $x^{\Delta\Delta} - 5x^\Delta + 6x = 0$ *on* $T = \Re$ in self adjoint form. Also express the difference equation x(t+2)- 5 x(t+1)+6 x(t)=0 on T=Z in self adjoint form.
6. Let A $\in \Re$ be a 2x2 matrix valued function and assume that X is a solution of $X^\Delta = A(t)X(t)$, then prove that X satisfies Liouville's formula.
7. State and prove Lyapunov inequality.
8. Solve the vector dynamic equation $x^\Delta = \begin{pmatrix} 3 & 1 \\ -13 & -3 \end{pmatrix} x$ for any time scale T.

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Code: 15 MAT 315

KL UNIVERSITY
DEPARTMENT OF MATHEMATICS
Pre Ph.D. Examinations
DIFFERENCE EQUATIONS

Eight questions are to be set and the student has to answer five in three hours of duration:

UNIT-1 : **Linear difference equations:** first order equations, general results for linear difference equations, solving linear difference equations.

Unit-2 : **Methods of solving linear difference equations:** solving linear difference equations with variable coefficients, nonlinear equations that can be linearized, solving difference equations using z-transforms.

(Scope and treatment as in chapter-3 of Text book (1))

Unit-3: **Linear initial value problems associated with system of difference equations:** introduction, preliminary results from algebra, linear dependence and independence, matrix linear systems, variation of constant formula, Green's matrix, systems with constant coefficients.

Unit-4: **Qualitative properties of solutions of difference systems:** continuation and dependence on initial condition and parameters, asymptotic behavior of linear and nonlinear systems.

Unit-5: **Stability of difference systems:** Concept of stability, stability of linear and nonlinear systems.

(sections 2.1 to 2.6, 2.7 and sections 5.1 to 5.6 of Text book (2))

Text Books:

1. Difference equations an introduction with applications by W. G. Kelley and A. C. Peterson
   Second edition, Harcourt Academic Press, USA (2001).
2. Difference equations and inequalities, theory, methods and applications by R.P.Agarwal,
   Baker publications, Marcel Dekker Inc, New York (1992).

KL UNIVERSITY
DEPARTMENT OF MATHEMATICS
Pre-Ph.D Degree Examination
MODEL QUESTION PAPER
DIFFERENCE EQUATIONS

Time: 3 Hours **Max.Marks:100**

Answer any FIVE questions from following, each question carries equal marks.

1. (a) Solve the difference equation y(t+1) - t y(t) = (t+1)!, t = 1, 2, 3,… so that y(1) = 5.

   (b) Solve the equation $u(t+1) = a\dfrac{(t-r_1)(t-r_2)...(t-r_n)}{(t-s_1)(t-s_2)...(t-s_m)}u(t)$, where $a, r_1,..., r_n, s_1,..., s_m$ are constants.

2. (a) Let $u_1(t), u_2(t)..., u_n(t)$ be the solution of $p_n(t)u(t+n) + p_{n-1}(t)u(t+n-1) + ... + p_0u(t) = 0$ and let w(t) be the corresponding Casoratian. Then show that

$$w(t+1) = (-1)^n \frac{p_0(t)}{p_n(t)} w(t).$$

   (b) Find all solutions of u(t+3) – 7u(t+2) + 16u(t+1) – 12u(t) = 0.

3. (a) Solve the difference equation $\Delta y(t) = 3^t \sin\frac{\pi}{2}t$.

   (b) Solve the equation $u(t+2) - u(t+1) - \dfrac{1}{t+1}u(t) = 0$.

4. (a) Solve the difference equation y(t+2)y(t) = y(t+1).

   (b) Solve the following initial value problem using z-transform
   (k+1)y(k+1)-(50-k)y(k) = 0, y(0)=1.

5. (a) Derive variation of constants formula for non-homogeneous difference equations u(k+1)=A(k)u(k) + B(k) satisfying $u(k_0)=u_0$.

   (b) Find the solution of the difference system u(k+1) = Au(k), where A = $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

6. (a) Let all solutions of difference system u(k+1) = Au(k) tend to zero as k → ∞. Then show that all solutions of u(k+1)=A u(k) + B(k) tend to zero as k → ∞ provided ||B(k)||→0 as k→∞.

   (b) Let for all $(k,u) \in N \times R^n$ the function F(k, u(k)) satisfy ||F(k, u)|| ≤ h(k) ||u||, where h(k) is a nonnegative function defined on N. Then show that

   (i) all solutions of u(k+1) = Au(k) + F(k, u(k)) are bounded provided all solutions of u(k+1) = Au(k) are bounded and $\sum_{\ell=0}^{\infty} h(\ell) < \infty$.

   (ii) all solutions of u(k+1)=A u(k) + F(k, u(k)) tend to zero as k → ∞ provided all solutions of u(k+1)=A u(k) tend to zero as k → ∞ and h(k)→0 as k→∞.

7. Define stable, uniformly stable and asymptotically stable and state and prove the necessary and sufficient conditions for stability, uniform and asymptotic stability of linear system u(k+1)=A(k)u(k).

8. State and prove sufficient conditions for the existence of stable solutions of nonlinear difference system u(k+1) = A(k)u(k) + F(k, u(k)).